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Precautionary Logic and Ignorance Interval Transfer Mediated Inference

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Abstract

A relatively formal framework for the description of application of the precautionary principle is described, which we consider to provide a basic setup for precautionary logic, that is reasoning patterns making use of said precautionary principle. Probability is understood as subjective probability in the tradition requiring exact figures for subjective probabilities. Besides probability we need various related concepts, notably: proposition, proposition space, truth, falsity, certainty, uncertainty, ignorance, hazard, harm, and risk. We adopt a fairly simple and coherent picture of these notions. We suggest that precautionary logic involves the transfer of quantified ignorance in the form of an ignorance interval.

Keywords: precautionary principle, precautionary logic, subjective probability

1 Introduction

This paper is a sequel to [2] where we have reviewed elementary aspects of subjective probability logic (as used in forensic logic) with a focus on the role of transfer of a likelihood ratio from (forensic) experts (together called MOE, an acronym for Mediator of Expertise) to judges or to the members of a jury (together referred to as TOF, an acronym for Trier of Fact) as a key step in

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the reasoning process. We will argue that the logic described in [2] may be referred to as likelihood transfer mediated subjective probability logic.

Legal reasoning is richer than what has been covered in [2] and in this paper we will discuss the situation arising if experts do not necessarily agree. The latter situation is frequently observed in cases of administrative law when application of the so-called precautionary principle are considered. Administrative law is under permanent development both on a national scale and on an international scale. A survey of such developments in the 20th century is given in Stewart [19]. In order to model disagreement between experts it is assumed that OMOE (opposing MOE) serves as an additional mediator of expertise.

Precaution is a complex theme. Following [12] we distinguish precautionary discourse, precautionary reasoning, precautionary logic, and precautionary inference. We propose that a dedicated inference rule may be helpful to capture the inference mechanism which seems to underly some applications of the precautionary principle in precautionary reasoning, thereby giving rise to a plausible precautionary logic.

The precautionary principle (PP) has many different wordings, to which we add the following description, where meta-uncertainty and ignorance are used as qualifications of defective or insufficient cognizance. In most wordings of PP the relevant lack of evidence is referred to as uncertainty, a choice of terminology which we consider to be less fortunate when used in a setting of subjective probability theory. PP-meta-uncertainty/ignorance is so named in order to highlight the particular use of terminology, meta-uncertainty and ignorance, notions the meaning and role of which will be discussed in detail in the paper.

Definition 1.1 *(PP-meta-uncertainty/ignorance) If planned actions A_{act} may conceivably induce a hazard H_{haz}, and estimating the risk coming with H_{haz} fails or is unconvincing because of meta-uncertainty concerning the probabilities involved (so that preventive logic does not apply), then ignorance (understood as lack of science based evidence) concerning causes and effects in relation to H_{haz} must not be taken for a ground to allow (or rather not to disallow) putting the plan to perform actions A_{act} into effect.*

We assume that precautionary logic may sometimes be used when preventive logic (prevention of hazards by way of risk analysis) is insufficiently informative.

We have made an attempt to identify a specific reasoning pattern, unavailable in likelihood transfer mediated subjective probability logic, as a

characteristic instance of precautionary inference.

By taking subjective probabilities as a point of departure we cannot escape the innate problems which render subjective probabilities difficult to use, such as the fundamental problem of how to arrive at plausible prior odds. Such worries need being taken care of and we assume that precautionary logic is used in cases where sufficient time is available to TOF to make up their mind on such critical matters.

Claim 1.1 *Ignorance interval transfer (from OMOE to TOF) for a likelihood constitutes a specific phase for precautionary inference, which thereby marks an extension of likelihood transfer (from MOE as well as from OMOE to TOF) mediated subjective probability logic.*

Equipped with an additional inference rule (below referred to as PIR-likelihood-ignorance-interval-transfer) a reasonable form of precautionary logic can be designed:

Claim 1.2 *The extension of likelihood transfer mediated subjective probability logic with likelihood ignorance interval transfer gives rise to a plausible precautionary logic based on subjective probability theory.*

A key aspect of subjective probability logic and Bayesian inference, is that reasoning takes the form of a process which proceeds in time, rather than the construction of a timeless proof, the well-known situation for mathematical proofs. We assume that precautionary logic, the application of the precautionary principle in reasoning is also best seen as a stepwise process.

We will frame precautionary logic as an extension of the logic described in [2], where the extension involves one additional rule: PIR-likelihood-ignorance-interface-transfer (with PIR abbreviating Precautionary Inference Rule). In the setting of precautionary logic it is assumed that there is a second mediator of expertise named OMOE , who may put forward to TOF views in disagreement with the views communicated by TOF to MOE. In the same manner as MOE, OMOE influences the process by transferring information to TOF.

A significant complication in comparison with [2] comes about from the fact that writing about precautionary logic and applications of the precautionary principle makes use of a richer language, involving notions for which the meaning is hardly standardized. These notions include: certainty, uncertainty, scientific uncertainty, degree of uncertainty, risk. We will propose a coherent view on the meaning of these terms which allows a concise and

convincing framing of precautionary logic. There may be relevant novelty in the package of interpretations of the various terms thus obtained. We are unaware of relevant references for said package.

Admittedly for each of these terms there is a substantial literature both proposing and surveying different meanings. Below a choice is made for each of these terms, in the awareness that other options exist, though with the objective to end up with a complete picture suitable for the task of describing (a form of) precautionary logic.

1.1 Preparatory Conceptual Design Decisions

We wish to detach the discussion from the complexities of international law. Many papers (see e.g., [18]) discuss the role and status of the precautionary principle in various jurisdictions, as well as the complications coming about from the range of different definitions thereof. An account of precautionary logic ought to stay outside such complexities. We will use precautionary logic as a label for logics involving one or more precautionary inference rules. Precautionary principles, of which there may be several are guidelines indicating that in certain circumstances the use of some form of precautionary logic may be used, may be necessary, or must be used. By default (see [21]) "the" precautionary principle refers to a guideline for reasoning in the context of, environment, ecology, climate, and various forms of impact of human behaviour on human health.

A precautionary logic is a logic which involves one or more precautionary inference rules. Precautionary logics are tools for analysing, understanding, and implementing precautionary principles. The focus of this paper is to formulate a precautionary inference rule and to indicate how the rule can be introduced in an existing subjective probability logic.

1.2 Proposition Space, Probability Function, Conditional Probability

It is assumed that all agents, X involved cast their views in terms of their beliefs in various propositions of a private proposition space S_X with logical connectives, $\neg, \wedge, \vee, \rightarrow$ and constants T and F. Beliefs are expressed in terms of a (subjective) belief function on S_X which is, due to the a probabilistic interpretation of belief, in mathematical terms a probability function on S_X.

We define a belief as a pair (A, r) with A a proposition (what is believed to be tru e) and $r \in [0, 1]$ the corresponding degree (to what extent is it

believed) the belief denoted as a probability. We use the convention that likelihood and conditional probability are synonyms. Both refer to a pair (t, r) with t a fracterm of the form $\frac{P(A \wedge B)}{P(B)}$ for some propositions A and B and r a real value for t. We refer to t as the likelihood expression and to r as the likelihood value. A likelihood ratio, is a pair (t, r) with r the value of t, and t a fracterm of the form $\frac{P(A|B)}{P(A|\neg B)}$.

Beliefs are the subject of subjective probability logics. For an accessible account thereof we suggest [13].

1.3 Knowledge Base of TOF and MOE

TOF as well as MOE and other agents of relevance maintain a database of facts where facts are items with belief 1 by definition. These items are not maintained in a belief function because there will be no probabilistic transformations. During an interactive reasoning process TOF must know which motivation MOE has delivered for its various transfers of probabilities, conditional probabilities and likelihood ratios. TOF must also keep in mind which transfers took place because probabilistic transformations must not be duplicated.

1.3.1 Database of Facts Versus Store of Beliefs

Besides the database of facts TOF maintains a belief function. In practice this will require automated support because transformations are hard to compute, and because valid probabilistic reasoning is hardly a built-in feature of the human mind. Starting from $P_{TOF,initial}$, which TOF must make explicit and store in their database, TOF maintains P_{TOF}. Different strategies are possible where TOF may or may not store and remember intermediate versions of P_{TOF}. We will assume that intermediate versions are all kept in memory.

1.3.2 Transformations of Belief

Agents change their beliefs on the basis of information transferred to them by other agents. A survey of transformations of the proposition space is given in [2]. The reasoning process of an agent takes one or more phases, where each phase is split up in one or more steps, the last step involving an update of the belief function by way of an appropriate transformation thereof. The survey of transformations of probability functions given in [2]

comprises: Bayesian conditioning, Jeffrey conditioning, single likelihood Adams conditioning and double likelihood Adams conditioning, plus several forms of propositions space reduction and of proposition space expansion.

We will mainly refer to single likelihood Adams conditioning, a transformation which, given a likelihood $(P(E|H), l)$ with $l > 0$ transforms P to $\widehat{P}$ given by

$$\widehat{P}(\phi) = P(H \wedge E \wedge \phi) \cdot \frac{l}{P(E|H)} + P(H \wedge \neg E \wedge \phi) \cdot \frac{1-l}{P(\neg E|H)} + P(\neg H \wedge \phi)$$

For details concerning both forms of Adams conditioning we refer to [2]. Here we have used E (with role of evidence) and H (with role of hypothesis) to have a fit with reference [2], below we will use other names for propositions, and H is reserved for the entropy function, as is conventional.

2 A Package of Terminology Conventions

We will adopt various conventions regarding the meaning of well-known terms. This is done in order to obtain a coherent framework which fits with the framework of subjective probability logic. The key decision taken here is to view probabilities as degrees of certainty/uncertainty. Lack of knowledge about probabilities is termed meta-uncertainty, or in specific cases such lack of knowledge is referred to as ignorance.

2.1 On the Plurality of Terminology Packages: Unavoidable Arbitrariness

A terminology package must be understood as no more than a mere choice for a combination of conventions about the meaning and use of terms, a choice which is made out of many alternative options for such combinations. For instance the philosophy of uncertainty in no way dictates that uncertainty is graded in two different ways, degrees quantified by way of probability, and levels which in turn split in metric levels and entropic levels. We use such conventions because we expect these to be helpful for a systematic description of a specific form of precautionary logic which is the focus of this paper. The terminology package outlined below is to some extent arbitrary, and such arbitrariness is unavoidable.

The terminology package chosen below is bottom-up in the following sense. It is assumed that initially one works with propositions (understood

with some some syntactic bias) for which only conjunction, disjunction, implication, and negation are available as connectives, in addition to a notion of truth. Doubt on truth introduces the need for falsity and the classical logic where writing propositions is independent of assessment of validity. At the same time one may notice the dropping of an initial bias: proposition is used without regard to truth, though in mathematics propositions are supposed to be true just as theorems are.

Then uncertainty arises as a next phase of doubt, and subjective probability calculus comes into play to quantify degrees of certainty, as well as degrees of uncertainty. $P_X(A)$ denotes the degree of certainty of X about a proposition A, alternatively referred to as the strength of belief of X in A. Similarly $1 - P_X(A)$ denotes the degree of uncertainty of X about A, alternatively referred to as the degree of disbelief of X about A.

A low degree of certainty for a proposition A correlates with a high degree of uncertainty for A, and also with a high degree of certainty for $\neg A$ and with a low degree of uncertainty for $\neg A$. In this case a bias is adopted rather than dropped as above: uncertainty about A is first of all understood as uncertainty about truth of A, and certainty about A is correspondingly understood as certainty about the truth of A.

However, the bias just mentioned is not unproblematic, and for that reason we consider besides the notion of a degree of uncertainty also the notion of a level of uncertainty, where A and $\neg A$ have the same level of uncertainty. Quantification of the level of uncertainty is first of all achieved with Shannon entropy, but there are other options available for that purpose.

Next it is observed that there may be a lack of insight/knowledge/awareness of the degree of uncertainty of A. Now meta-uncertainty enters the picture, which in some cases is referred to as ignorance. In the simplest case meta-uncertainty is quantified in terms of ignorance intervals. (We notice that such intervals have been studied in great depth, for instance in [1].)

2.2 Risk, Risk Factor, Risk Factor Probability, Risk Factor Cost

The use of the precautionary principle is often motivated in terms of risk management. Terminology about risk has not been fixed once and for all, however. We propose to speak of risk factor (also termed hazard), risk factor probability, risk factor cost, and of risk, these being quite different notions. First of all risk factor describes the quality of a risk, i.e. what happens, if it happens (for instance the risk that the sun explodes and thereby destroys

most life on earth). We say that a risk factor becomes active in case it takes place, or in other words comes about, or materializes. Risk factor cost (also termed harm) expresses the expected cost conditional on the risk factor having become active. The risk factor probability is the probability of the latter, i.e., the probability that the risk factor materializes. Risk, on the other hand is a quantity, i.e., is in principle understood as an expected value determined as risk factor probability multiplied with risk factor cost. Risk therefore has a dimension (one or more units of cost) and a quantity (a vector of rational numbers) counting/describing the size of the risk as a multidimensional entity of the given dimension.

2.3 Certainty and Uncertainty

We assume that all agents maintain a probability function on a relevant proposition space S. For agent X we will denote the probability function with P_X and if initial, prior, posterior (both relative to a step s in an argument involving one or more steps) or final probability functions are meant we may write $P_{X,initial}$, $P_{X,prior,s}$, $P_{X,posterior,s}$ and $P_{X,final}$. Propositions are named $A, B, ...$ Now we understand certainty and uncertainty as follows:

- X is certain about A if either $P_X(A) = 0$ or $P_X(A) = 1$,
- X is certain about the truth of A if $P_X(A) = 1$,
- X is certain about the falsity of A if $P_X(A) = 0$,
- X is uncertain about A if X is not certain about A,
- $P_X(A)$ denotes the degree of certainty of X about A, $P_X(\neg A) = 1 - P_X(A)$ denotes the degree of uncertainty about A.

2.3.1 Level of Uncertainty

While we understand the degree of certainty of a proposition A as a measure of belief that A is true, we adopt level of (un)certainty as a different notion from degree of (un)certainty.

- X the metric level of uncertainty of X about A is $\min\{P_X(A), 1 - P_X(A)\}$ (we use the qualification metric in the light of the availability of entropy as a more sophisticated quantification of uncertainty),

- The level of uncertainty of a probability distribution P is measured with Shannon entropy $H(P)$ which pertains to the entire probability function P, where we follow [17] on this matter.

2.3.2 Fracterm Calculus: Entropic Transreals Instead of Suppes-Ono Arithmetic

We understand entropy as a measure of uncertainty, thus following e.g. [17]. Specifically, given a belief function (probability function understood by way of subjective probabilities) P, the entropy $H(P)$ serves as a quantification of the level of the uncertainty embodied in P. Reasoning in [2] was based on the Suppes-Ono convention for totalizing division: adopting $1/0 = 0$. It was elaborated in [8] that, when entropy and cross-entropy are to be used and defined, another convention for totalizing division is preferable: entropic transreals. Entropy matters for precautionary logic because it may be understood as a measure of the level of uncertainty.

When extending the subjective probability logic of [2] to precautionary logic where uncertainty plays a more prominent role, we will adopt entropic transreals rather than working with the Suppes-Ono convention (as is done in [2]). Entropic transreals are found as an enlargement of the field of reals where the domain is extended with three elements: ∞, $-\infty$ and $\perp$, and where the signature is extended with the sign function $\mathbf{s}$, totalized division, and base 2 logarithm.

Characteristic facts for entropic transreals are these: $\frac{1}{0} = \infty$, $\frac{1}{\infty} = 0$, $0 \cdot \infty = 0$, $\infty + (-\infty) = \perp$, $\infty + \infty = \infty \cdot \infty = \infty$, $\log_2 0 = -\infty$, $\log_2 1 = 0$, $\log_2 2 = 1$, $\log_2 4 = 2$, ..., and $\log_2 -1 = \log_2 -2 = .. = \log_2 -\infty = \perp$. For the sign function we have: $\mathbf{s}(0) = 0, 1 = \mathbf{s}(1) = \mathbf{s}(2) = ..., -1 = \mathbf{s}(-1) = \mathbf{s}(-2) = .., \mathbf{s}(\infty) = 1, \mathbf{s}(-\infty) = -1$ and $\mathbf{s}(\perp) = \perp$. We denote entropic transreals with $\mathbb{R}_{\infty,\perp,\mathbf{s},\log_2}$ and the minimal subalgebra thereof of with $\mathbb{Q}_{\infty,\perp,\mathbf{s},\log_2}$. $\mathbb{Q}_{\infty,\perp,\mathbf{s},\log_2}$ is the primary datatype in which calculations and quantified assessments for precautionary logic may be performed. Remarkably little is known about this datatype, for instance we have no answer to the following question:

Problem 2.1 *Is the datatype* $\mathbb{Q}_{\infty,\perp,\mathbf{s},\log_2}$ *computable?*

The issue is far from obvious. It is neither easy to see that $\mathbb{Q}_{\infty,\perp,\mathbf{s},\log_2}$ is semi-computable, nor that $\mathbb{Q}_{\infty,\perp,\mathbf{s},\log_2}$ is co-semicomputable (as defined e.g. in [7]).

Almost without modification the results of [2] carry over to a setting of entropic transreals.

2.3.3 Level of Certainty of a Proposition

We understand $\frac{1}{H(A)}$ as a level of certainty for A. In entropic transreals this convention produces $+\infty$ as the level of certainty for a probability distribution which features probabilities 0 and 1 only (in other words a probability function which expresses certainty about all propositions involved).

We recall that for a single proposition A and an agent X we distinguish: degree of disbelief (= degree of uncertainty), metric level of uncertainty, and (entropic) level of uncertainty.

Similarly we distinguish: degree of belief (= degree of certainty), metric level of certainty, and level of certainty.

The proposed terminology is complicated by the fact that we insist that certainty and uncertainty are first of all understood as "complications" which trigger and motivate the use of probability theory, and doing so in such a manner that probabilities are used to quantify certainty and uncertainty (of the truth of a proposition in the mind of an agent). While probability expresses a degree of (un)certainty, we need another label for the quantification provided by entropy: we propose that entropy quantifies level of uncertainty rather than degree of uncertainty.

2.3.4 Probability Calculus in Entropic Transreals

A probability function on a proposition space S takes values in $[0,1]$. Conditional probability $P(A|B)$ is defined as $\frac{P(A\wedge B)}{P(B)}$. A conditional probability takes values in $[0,1]$ as well. Further if $P(B)=0$ it must be the case that $P(A\cap B)=0$ so that $P(A|B)=\frac{0}{0}=0\cdot\infty=0$. If $P(A)=0$ then $P(A|B)=\frac{0}{P(B)}=0\cdot\frac{1}{P(B)}=0$, as $0\cdot x=0$ for all $x\neq\bot$.We find that $P(A|B)=0$ if and only $P(A)=0$ or $P(B)=0$. With entropic transreals, Bayes' theorem can be formulated without any non-zeroness conditions on probabilities, just as when working with the Suppes-Ono convention:

$$P(A|B)=\frac{P(B|A)\cdot P(A)}{P(B)}$$

Jeffrey conditioning serves to revise a belief function P in order to ensure that $P(A)=p$. The new belief function $P^J_{A,p}(\phi)=p\cdot P(\phi|A)+(1-p)\cdot P(\phi|\neg A)$

works well, also for $p = 0$ and for $p = 1$ where in the latter case it works just as Bayes conditioning: $P_A^B(\phi) = P(\phi|A)$.

A likelihood ratio is a value in $[0, \infty]$. Likelihood ratios may be arbitrarily large including the infinite value, for instance with $P(A) = 1/2$, one finds $LR(A, A) = \frac{P(A|A)}{P(A|\neg A)} = \frac{1}{0} = \infty$.

While involutive meadows, like fields, though with division and using the Suppes-Ono convention allow a nice equational axiomatization, which cover probability calculus as well (see [3]), finding an axiomatization for which a plausible completeness result can be found is much harder for entropic transreals. For the purposes of subjective probability theory the absence of a complete axiomatization is not a worrying roadblock, however, because when needed one may use weaker and incomplete axiom systems. The various results of [2] can be trivially reformulated in the context of entropic transreals, and proofs can be used without change.

2.4 Meta-uncertainty and Quantified Meta-uncertainty in the Absence of Meta-certainty

When probabilities are in doubt some form of lack of knowledge enters the picture rather than uncertainty. Absence of convincing knowledge may take different forms and we will first of all use meta-uncertainty as a label for uncertainty about a probability in a probability distribution. A meta-uncertainty interval for a proposition A (and for an agent X) is a non-empty interval $[a, b] \subseteq [0, 1]$ with the understanding that X knows no better than that the probability of A being true lies in the interval $[a, b]$. It is plausible that $P_X(p) \in [a, b]$. We may measure meta-uncertainty about A, if present, as $b - a$ (taking $\perp$ for its value otherwise).

At this point we deviate from subjective probability theory: X may have in mind a meta-uncertainty interval for A, and X may even have incoherent views in case $P_X(A) \notin [a, b]$. Not all awareness of certainty/uncertainty (for X about A) is encoded in the belief function P_X. We understand meta-certainty, for X about A, as the absence of a meta-uncertainty interval regarding A in the mind of X.

2.4.1 Ignorance, Ignorance Interval

In some cases meta-uncertainty may be qualified as ignorance. First of all ignorance is significant meta-uncertainty. Here significance is an informal qualification which is context dependent. Ignorance has a negative conno-

tation, however, which we consider adequate in the case of precautionary logic.

Definition 2.1 *Ignorance (of agent X, regarding proposition A) is imputable lack of meta-certainty (about A), or stated alternatively an imputable (in the light of available information regarding the objectives of X) level of meta-uncertainty (about A).*

The use of ignorance as a label for the alleged lack of information of X about A is not merely justified by X's potential missing information. Ignorance is also used as a qualification in order to express that for the application at hand in which X makes use of, or might make use of, say $P_X(A) = r$, according to some other agent Z, X should be less ignorant and should be (or have been) clearly able to narrow down the (claimed) meta-uncertainty interval to a much smaller neighbourhood of r, thereby increasing the plausibility of meta-certainty concerning their certainty about the truth of A.

It may be so that $P_X(A) = r$ and that X justifies certain behaviour or views on that state of affairs. (As an example of such behaviour we will consider the case that X gives another agent, say Y permission to perform activity ACT_Y.) Now another agent, say Z may claim that X lacks the meta-certainty which is supposedly needed to adopt whatever subjective probability r as their level of certainty that A is valid, thereby overruling any arguments that may have led X to adopt $P_X(A) = r$.

2.5 Belief Hiding for TOF, Though not for MOE and for OMOE

It is an underlying assumption, in subjective probability calculus as used say in subjective probability logic with likelihood transfer mediation, of the brand discussed in [2], that TOF privately maintains their belief function P_{TOF}, so that, if $P_{TOF}(A) = r$ only TOF knows that such is the case. From this observation it may be concluded that an agent who indicates an ignorance interval about A will do so without making any reference to r. This observation will be reflected in the design of the inference rule PIR-likelihood for the extension from a logic with likelihood ratio transfer based inference to precautionary logic.

Belief hiding is one of several assumptions underlying likelihood ratio transfer based logic. Another assumption is that MOE (and OMOE when present) need not adopt subjective probability theory and are at liberty to determine the value of relevant parameters on the basis of a frequentist

appreciation of probability and statistics. It is up to TOF to make sense in terms of subjective probabilities of the information transferred to them by MOE or by OMOE.

2.5.1 Framework Openness for TOF, Though not for MOE and for OMOE

For the work in this paper it is assumed that TOF adopts subjective probability theory as their conceptual framework and that this state of affairs is publicly known, and that TOF makes systematic use of subjective probabilities. However, more generally, and in practice TOF may not do so. We hold that in general TOF must be public about their appreciation of probability theory: are they adopting subjective probabilities as the leading framework or otherwise. As stated, here it is assumed that TOF has publicly announced its adoption of subjective probabilities. There is no requirement that MOE and if present OMOE are public about their adoption of a framework for probability calculus and probabilistic reasoning.

These issues are discussed in some detail in [2]. MOE and OMOE need not hide their view of probability theory and they may even claim disagreement about such matters with TOF (who supposedly adopts subjective probability in a fairly strict sense).

2.5.2 Reversibility Requirement

Every revision of the probability function of TOF must be performed in such a manner that it can be reversed or at least that subsequent revision for all propositions involved is still possible. Reversibility first of all implies that reduction of the proposition space is disallowed.

The reversibility requirement serves to guarantee that whenever MOE communicates some degree of certainty about a proposition A the corresponding level of certainty is never equal to $+\infty$, so that there will be some remaining uncertainty, and so that in a subsequent step new information coming from MOE or coming from an opposing OMOE, may change the probability distribution once more in such a manner that the probability of A is revised. Technically reversibility implies that Bayesian conditioning is not used, instead a high non-unit probability must be chosen and Jeffrey conditioning must be applied. Further it is assumed that the likelihood which is communicated never equals 0 or 1 and the likelihood ratio never equals 0 or $+\infty$. In both cases this requirement is imposed in order to allow

subsequent updates of the (un)certainty information relevant for the case. The issue is that Bayesian conditioning cannot be reversed while Jeffrey conditioning, for a probability in the open interval $(0, 1)$ can be reversed.

3 Subjective Probability Logic with Likelihood Transfer Mediated Inference

The objective to extend the logic of [2] provides an incentive to simplify the naming conventions of the latter. We will refer to the logical system outlined in [2] as a subjective probability logic (of which there are many), with an inference mechanism based on the transfer of conditional probabilities.

TOF maintains a belief function P_{TOF} on proposition space S, and in a stepwise reasoning process which involves a sequence of phases, where each phase has a similar structure:

a) (Transfer step) MOE indicates an aspect of the proposition space about which they may have something new to say, plus a new value for it. This aspect is among one of three options: (i) a proposal for a new probability of a proposition in the proposition space, (ii) a proposal for a new value for a likelihood over the proposition space, (iii) a proposal for a new value for a likelihood ratio over the proposition space. MOE indicates the syntactic expressions which this is about, that is for case (i) a proposition, for case (ii) a fracterm for a likelihood, and in case (iii) a fracterm for a likelihood ratio.

b) (Motivational step) MOE produces a motivation for its proposal of a new value for a belief, a likelihood, or a likelihood ratio.

c) (Decision taking step) TOF takes a decision about whether or not to accept the new proposal.

d) (Transformational step) If TOF has taken the decision not to accept the proposal of MOE, the belief function of TOF remains unchanged, otherwise if TOF accepts the proposal, an update is made. Updating works as follows: the belief function P_{TOF} is transformed:

   d1) if (P, r) is a proposed revision of belief, then there are three cases:
   - if $r = 1$ then Bayesian condition w.r.t. P is used;
   - if $r = 0$ then Bayesian conditioning w.r.t. $\neg P$ is used;

- if $0 < r < 1$ the Jeffrey conditioning w.r.t. P is used;

d2) if $(P(A|B), r)$ is a proposal for a revision of the value of a likelihood, then it may be assumed that $r \neq 0$ and the belief function is updated by means of single likelihood Adams conditioning;

d3) if $(\frac{P(A|B)}{P(A|\neg B)}, r)$ is a proposal for a revision of the value of a likelihood ratio, say then $P(A|\neg B) \neq 0$ and $P(A|B) \neq 0$ and the belief function is updated by means of double likelihood Adams conditioning.

3.1 A Simplification: Restriction to Likelihood Transfer

Now one may notice that Bayesian conditioning is a special case of Jeffrey conditioning so that Bayesian conditioning need not be treated as a special case. Further we notice that an update (A, r) is equivalent to an update $(P(A|\mathsf{T}), r)$ so that proposals for updating a belief may be replaced by proposals for updating a likelihood. If the proposal is made for an update of a likelihood ratio $(\frac{P(A|B)}{P(A|\neg B)}, l)$, then it may be assumed that MOE knows values, say l_1 and l_2 for $P(A|B)$ and $P(A|\neg B)$ respectively and, following the results of [2] by applying first a single likelihood Adams transformation w.r.t $(P(A|B), l_1)$ followed by single likelihood Adams conditioning w.r.t. $(P(A|\neg B), l_2)$ the same result is obtained as applying a double likelihood Adams conditioning w.r.t. the likelihood ratio $(\frac{P(A|B)}{P(A|\neg B)}, l)$.

In case MOE proposes a new value for a likelihood ratio $(\frac{P(A|B)}{P(A|\neg B)}, l)$ without any incentive to split the update of a likelihood ratio in two parts, we assume that MOE has two likelihoods l_1 and l_2 in mind so that it follows from the results of [2] that MOE may simply choose values l_1 and l_2 such that $l = \frac{l_1}{l_2}$, and such that proposing revisions $(P(A|B), l_1)$ and $(P(A|\neg B), l_2)$ can both be convincingly motivated to TOF. By successively transferring $(P(A|B), l_1)$ and $(P(A|\neg B), l_2)$, each in combination with convincing argument, single likelihood Adams conditioning may be applied twice, so that (according to the [2]) the same result is obtained as by way of double likelihood conditioning w.r.t. the likelihood ratio $(\frac{P(A|B)}{P(A|\neg B)}, l)$.

As it turns out the subjective probability logic with likelihood ratio transfer based inference of [2] can be simplified on the basis of the results in [2] so that instead of three key parameters for transfer: belief, likelihood and likelihood ratio, only likelihood transfer $(P(A|B), r)$ with $r \neq 0$ is needed.

3.2 Successive SPLR-likelihood-transfer Phases Suffice

We denote a phase where a likelihood (conditional probability) is transferred as a SPLR-likelihood-transfer mediated phase (where SPLR abbreviates subjective probability logic rule). It follows that each sequence of phases in SPLR may be seen as a sequence of phases for an SPLR-likelihood-transfer, so that a session with TOF and MOE consist of one or more SPLR-likelihood-transfer phases. This naming is introduced in order to achieve uniformity with the description of rules for precautionary inference. Once TOF is satisfied and can make up their mind a probability $P_{TOF,final}(R) = p$ is determined for a proposition R which expresses the resulting situation (for instance that a specific hazard has not taken occurred). We assume that TOF had a threshold p_{thr} in mind and if $p \geq p_{thr}$ TOF delivers a positive result (for instance granting permission for some specified actions), while otherwise, if $p < p_{thr}$ TOF delivers a negative result (for instance refusal to grant permission for some specified actions). The result of TOF is also called their verdict.

3.3 What is Missing?

Suppose that defendant X_{def} is suspected of having stolen a certain painting of Jan Mankes (at a given date, from a given location etc.). Jan Mankes (1889-1920) is a remarkable Dutch painter who, unlike Van Gogh, has not (yet) acquired international fame. Now assertion A states that indeed X_{def} has stolen the painting, and, starting from plausible prior odds, after having processed various bits and pieces of information from MOE (or several MOE's) TOF arrives at posterior odds $P_{TOF}(A) = p$ such that $p > p_{thr}$ (a predefined threshold), say $p_{thr} = \frac{9}{10}$ and on that basis concludes that A is true so that X_{def} is considered guilty and will somehow be punished for that reason.

One may imagine that in appeal another MOE, named OMOE (opposing MOE) comes forward with additional evidence thereby extending the case, assuming that MOE introduces that same arguments and transfers the same parameters as before. We also assume that in appeal TOF (which may have a different constitution, i.e. may consist of different agents) starts out with the same prior odds. OMOE may introduce an agent Y_{new} into the discussion, without X_{def} being present, or even aware of their involvement. OMOE may argue for proposition A' which states that Y_{new} has stolen the painting, and the argument may be so convincing that finally $P_{TOF}(A') > p_{thr}$, so that

TOF accepts A' as valid. Because $P(A|A') = 0$ it follows that A is invalid, and after all the conviction of X_{def} is undone in appeal.

3.4 A Remedy: Capturing Lack of Proof by Hypothetical Reasoning

In criminal cases, it is very demanding to prove that X_{def} is not guilty by proving that someone else is guilty. There is a notion "lack of proof". The explanation of subjective probability based reasoning involving parameter transfer (in particular likelihood ratio transfer) as given in [2] is uninformative about the notion of "lack of proof". However, an interpretation of lack of proof can be informally given as follows.

Lack of proof is established if an OMOE creates one or more ignorance intervals in the mind of TOF, as well as imagining a hypothetical agent Y_{new}^{hyp}, and an assertion A'' asserting that Y_{new}^{hyp} has stolen the painting, such that upon proposing (by OMOE) well-chosen parameters that lie inside these ignorance intervals, and transferring these parameters to TOF, TOF will infer that $P_{TOF}(A'') > p_{thr}$.

In criminal law also the latter mechanism is too demanding, as it deviates form the idea that in order to convict A, i.e., to infer the validity of A positive evidence is needed for that proposition, and that for arguing the absence of positive evidence there is no requirement to establish that in some hypothetical case negative evidence may be imagined. In administrative law the mechanism just described, with an OMOE transferring an ignorance interval plays a central role as one of several ways to make use of the precautionary principle.

4 Precautionary Logic with Ignorance Interval Transfer Mediated Inference

Precautionary logic with ignorance interval transfer mediated inference combines subjective probability logic with likelihood transfer mediated inference (as outlined above) with an additional inference mechanism based on the transfer of an ignorance interval. The latter phase may be followed by oppositional parameter transfer. The latter inference rule is named PIR-likelihood-ignorance-interval-transfer (precautionary inference rule mediated with transfer of an ignorance interval for a likelihood), and it works in detail as follows.

4.1 PIR-likelihood-ignorance-interval-transfer

The following describes a single reasoning phase s which consists of four steps. The phase is called PIR-likelihood-ignorance-interval-transfer.

1. (likelihood expression transfer) OMOE will provide the fracterm $P(A|B) \equiv \frac{P(A \wedge B)}{P(B)}$ thereby providing to TOF all relevant syntactic information concerning a potential/proposed belief revision.

2. (Ignorance interval transfer) OMOE transfers to TOF an ignorance interval $[a, b]$ where $a > 0$, with regard to the conditional probability $P(A|B)$, which TOF has just been made aware of. By transferring the ignorance interval OMOE claims to provide a better representation of what is known about $P(A|B)$ than adopting $\frac{P_{TOF,prior,s}(A \wedge B)}{P_{TOF,prior,s}(B)} = r$ as TOF does (irrespective of r, a value which is assumed not to be known to OMOE in view of belief hiding for TOF as mentioned above in Subsection 2.5).

3. (Motivation) Subsequently OMOE explains to TOF their grounds for claiming $[a, b]$ as an ignorance interval for $P(A|B)$. We notice that the larger $b - a$ is, the less trust OMOE has in any conceivable value for the subjective conditional probability $P(A|B)$ as maintained by TOF. It is up to OMOE to convince TOF of their lack of trust on the full range of possible values for $P(A|B)$, including a justification for the lower bound a and the upper bound b.

4. (Acceptance or dismissal of ignorance interval) TOF takes a decision: do they trust OMOE on the matter?

   - If TOF takes the decision not to trust OMOE, TOF dismisses the ignorance interval as transferred by OMOE and leaves their belief function unchanged, that is: $P_{TOF,posterior,s} = P_{TOF,prior,s}$, and TOF informs OMOE accordingly about their dismissal of the proposed ignorance interval.
   - Alternatively if TOF takes the decision to trust OMOE in this case, OMOE succeeds with the transfer of an ignorance interval and TOF responds accordingly to OMOE.

Having performed this phase and if TOF has taken the decision to trust OMOE, TOF has included in its database that it is willing to accept a transfer of likelihood $(P(A|B), r)$ for any value in the ignorance interval it has just

accepted. If TOF rejected the ignorance interval no such information is stored and OMOE will need "better" arguments when transferring a likelihood $(P(A|B), r)$.

4.2 PIR-oppositional-likelihood-transfer

Another phase for OMOE is PIR-oppositional-likelihood-transfer where OMOE takes the liberty to transfer to TOF a likelihood which matches.

1. (Oppositional parameter determination) OMOE determines a non-zero and non-unit value, say r for the conditional probability $P(A|B)$ which may be chosen at their own discretion by OMOE within the ignorance interval $[a, b]$ (so that $(r \in (0, 1) \cap [a, b])$. The parameter is considered oppositional because it is plausible that MOE disagrees and will part of their work reversed upon prospective belief revision by TOF.

2. (Oppositional parameter transfer and corresponding belief revision) OMOE then transfers likelihood $(P(A|B), r)$ to TOF, with the claim that it matches an accepted ignorance interval. The transfer is received by TOF and subsequently TOF (who remembers that it has accepted an ignorance interval $[a, b]$ for $P(A|B)$) adopts r as a convincing update for $P(A|B)$ and uses single likelihood Adams conditioning w.r.t. $P(A|B) = r$ to transform $P_{TOF,prior,s}$ to $P_{TOF,posterior,s}$ accordingly. If TOF has no information in store about acceptance of an ignorance interval, or if r lies outside the interval (or equals one of its boundaries) , TOF responds accordingly to OMOE and leaves its belief function unchanged.

4.3 Further Phases of the Session with TOF, MOE and OMOE

The result of a phase of PIR-likelihood-transfer is again a state where TOF maintains a probability function as before. Now subsequent phases either from subjective probability logic, that is phases mediated with likelihood transfer as described above as SPLR-likelihood-transfer phases, or PIR-likelihood-transfer phases may occur until TOF is satisfied with their interaction with MOE and OMOE and ends up with a belief function $P_{TOF,final}$. Just as in Subsection 3.2, TOF will produce a verdict.

5 Cautious Usage of Precautionary Logic

A major criticism, massively represented in the literature of precautionary discourse, claims that precautionary logic may produce problematic conclusions. The theoretical account given above provides no safeguards against such incidents. We will describe an informal protocol for a canonical case: an appeal about a permission which has been granted. The example below serves two objectives. First of all application of PIR-likelihood-ignorance-interval-transfer as a step in an inference process makes sense only in practice in a more involved context, and the example introduces elements of such context. Secondly safeguards have been built in, in order to avoid implausible conclusions.

5.1 A Cautious Protocol for Using Precautionary Logic in an Appeal of Granted Permissions

Suppose permission is requested for action A. Permission may be refused on the basis of the combination of grounds listed below. Here we assume that it is feasible to apply hypothetical reasoning under the hypothesis that permission is granted and A is put into effect.

1. There is a requesting agent $\mathsf{X}_{\mathsf{asking}}$ who asks for permission. There is a protesting agent $\mathsf{X}_{\mathsf{protesting}}$ who asks for refusal of granting permission.

2. The dispute may be understood as a conflict between $\mathsf{X}_{\mathsf{asking}}$ and $\mathsf{X}_{\mathsf{protesting}}$. The dispute comes about after $\mathsf{X}_{\mathsf{asking}}$ had been granted permission for A. TOF serves as judge. MOE produces input for phases for $\mathsf{X}_{\mathsf{asking}}$, while OMOE produces input for phases for $\mathsf{X}_{\mathsf{protesting}}$.

3. There is a risk, say R, consisting of a chain of events which might be caused by A such that eventually a condition H (for hazard) comes about. (The mere possibility of this chain of events needs to be plausible, the risk combines the seriousness of the hazard, that is the harm done when it occurs, with the probability of its occurrence).

4. Condition H is irreversible, and its coming about may take place with significant delay so that "wait and see" (permit action A as long as condition H does not come about) is not a satisfactory policy for agent $\mathsf{X}_{\mathsf{protesting}}$.

5. If it were certain that action A causes condition H to come about then that state of affairs (i.e., that certainty) provides sufficient grounds for not granting permission for action A.

6. (Application of subjective probability logic) There is evidence available that with a probability at least $p_{rknown/min}$ action A when performed will eventually lead to condition H.

7. (Application of subjective probability logic) The precondition of item 5 is not satisfied. More specifically there is evidence that the probability of condition H being caused by actions A is below a certain probability $p_{rknown/max} < 1$.

8. (Arithmetic) $p_{rknown/min} < p_{rknown/max}$, in other words available research data are consistent. Moreover, both $p_{rknown/min}$ and $p_{rknown/max}$ have been established with the same or comparable mechanisms concerning the causal chain from A to H in mind.

9. (Policy decision) There is a rational number $p_{r/block}$ with $p_{rknown/min} < p_{r/block} < p_{rknown/max}$ such that: if it were likely with probability $p > p_{r/block}$ that action A causes condition C to come about, then that state of affairs provides sufficient grounds for not granting permission for action A. (i.e., from probability $p_{r/block}$ upwards risk R is considered too high for granting permission for A).

10. (OMOE makes up their mind about an ignorance interval) A panel (working as OMOE) of scientists representing both those who found evidence for the determination of $p_{rknown/min}$ and those who found evidence for the determination of $p_{rknown/max}$ agree (as scientists asked for an opinion on the matter knowing that there is a lack of evidence) that (A) below is more plausible than (B).

    (A) further research might (when performed and on the long run) reveal that a better lower bound $p'_{rknown/min} \geq p_{r/block}$ can be found, (in which case permission could be refused on the basis of available evidence),

    (B) further research might (when performed and on the long run) reveal that a better (i.e., sharper, lower) upper bound $p'_{rknown/max} < p_{r/block}$ can be found (so that permission could be granted on the basis of available evidence).

11. (OMOE considers the case from first principles) The panel mentioned in item 10 above has been convened for this specific case, in particular the conclusions of the panel are not a matter of plausible inference from panel conclusions for previous cases. (The import of potentially inconclusive results of preceding legal procedures must be prevented).

12. (OMOE suggests a parameter) OMOE suggest to TOF that $P(H|A)$ might be equal to $p^\star \in (p_{r/block}, p_{rknown/max})$, with the effect that a PIR-likelihood-transfer phase has been completed so that upon belief revision TOF will withdraw the granted permission.

13. (Use of precautionary inference is essential, other solutions are preferred) The conflict has no economic solution: there is no financial (or otherwise economic) solution in either direction: an amount which one side may be asked to pay to the other side in the conflict so that the paying side gets the outcome their way. Moreover: a study has been made on how such economic solutions might look like in this particular case and why such solutions will not work.

    If an alternative solution, without usage of precautionary inference, for the conflict can be found that solution is to be preferred.

14. The precautionary inference application penalty is made explicitly aware and is being taken care of. This penalty involves at least the following elements:

    (i) Reopening the case on request of (permission requesting) agent $\mathsf{X}_{\mathsf{asking}}$ (after permission has been refused) is unproblematic for $\mathsf{X}_{\mathsf{asking}}$ once new evidence is available.

    (ii) An open invitation to the research community to provide comments on the methodology of the case and the merits of its outcome.

    (iii) A mechanism for revision of the case triggered by such comments is available, even if requesting agent $\mathsf{X}_{\mathsf{asking}}$ has lost interest or has ceased to exist as an active side in the conflict. Such revisions come at no cost for $\mathsf{X}_{\mathsf{asking}}$, and may be charged on $\mathsf{X}_{\mathsf{protesting}}$.

5.2 Commenting the Protocol

Comments on the description of a cautions protocol for using precautionary logic in an appeal of granted permissions.

1. Condition 7 is adopted in order to guarantee that there is some established practice of management of the risk involved, which in turn should guarantee that the risk is not merely imagined.

2. Condition 2 excludes certain important situations, for instance global warming and its consequences are risks of a different form, because such risks that involve a hazard which is not conceivably caused by the actions of a single agent $\mathsf{X}_{\mathsf{asking}}$.

3. Condition 6 is included to make sure that the risk involved is realistic. A probabilistic approach to its likelihood is feasible.

4. Condition 7 implies that it is known that the premise of item 5 above is not valid (so that item 5 cannot be used as a decision rule in this case).

5. Condition 13 must be established by way of various proposals for an economic solution and reasons for rejection of these. In particular, if the presence of the risk (even before coming into effect) can be measured in terms of financial cost, a solution along such lines is preferable.

6 Concluding Remarks

The negative bias of ignorance as used in precautionary logic comes about from the duty agent TOF is supposed to have to defend its conditional beliefs, say $P(A|B)$ against attacks on the chosen value r for the subjective conditional probability $P_{TOF}(A|B)$ which TOF has adopted. OMOE by transferring an ignorance interval indicates to TOF a suspicion of unwarranted belief and this suspicion is phrased in terms of (suspected) ignorance.

6.1 Ignorance Versus Uncertainty; Cognizance and Non-cognizance

The terminology of certainty, uncertainty, meta-uncertainty and ignorance lacks notions which one may need in different circumstances. For instance if a decision needs to be taken between proceeding in two different ways there may be probabilistic information available which allows to attach expected utilities to both options. Alternatively no such information may be available, for instance because of significant meta-uncertainty about the effects of both

actions, and that meta-uncertainty may not at all be a failure of the decision taking agent. We propose to speak in such a situation of decision taking in condition of non-cognizance. Non-cognizance involves an awareness of meta-uncertainty.

In some cases, however, non-cognizance may be blamed on an agent who initially claims cognizance. Then we speak of ignorance, which thereby is an instance of non-cognizance under specific conditions. Ignorance about a proposition A can be expressed via an ignorance interval $[a, b]$ with $a \leq b$, which indicates, for an agent P that the probability p of a proposition A lies between a and b. Ignorance is present in case $a < b$. Cognizance is an opposite situation of ignorance, non-cognizance is substantial meta-uncertainty, without any bias that the meta-uncertainty is in any way blamable.

Conversely: cognizance implies the absence of ignorance. An agent is cognizant about some probability function on proposition space S if they are not ignorant about any proposition in S, There are many views on how to incorporate ignorance in a setting of subjective probability theory, ranging from outright rejection of the concept to the use of second order probability distributions (probabilities of probability functions). The concept of ignorance is at odds with strict subjective probability theory and at the same time the idea is appealing, practical and hard to refute.

We will take ignorance, once admitted, for the awareness that one's subjective probability function may be about to change and an ignorance interval provides guidance about the scope of that change. Ignorance is a state of being open minded for adopting another agent's beliefs on some matters. This openness is connected with trust, only trusted agents may enact a change of degree of (conditional) belief (in other words a change of certainty/uncertainty) about a conditional proposition.

For instance an agent X may be certain that "$1 + 1 = 2$" is a valid (true) proposition. There is no uncertainty, either, however, as X may not in any way think of the probability of "$1 + 1 = 2$" as being about to change through becoming better informed by a trusted agent. So one may assume that for agent X there is no uncertainty about $0 = 1$, it is false.

However, having been informed that mathematics (ZF, Peano, or whatever) might be inconsistent, X may briefly adopt an uncertainty interval $[0, -\epsilon]$ for $0 = 1$. It is plausible that ignorance gradually disappears in the mind of an agent. Not having reflected about the possible collapse of known mathematics X may return to adopting the trivial ignorance interval $[0, 0]$ for the proposition stating that $0 = 1$.

6.2 Literature on Precautionary Logic

The term precautionary logic has been advocated by various authors, who adopt the view that precautionary logic is informal reasoning making use of a precautionary principle. We are unaware of any formalization of precautionary logic. Arnoldussen [1] provides a historic overview of the precautionary principle, advocating it as a mechanism for inducing moderate behaviour. Trouwborst [21] distinguishes preventative logic from precautionary logic. Following [21] the rise of use of a precautionary principle is linked to recent forms of awareness of both the gravity and the unpredictability of ecological problems. The distinction between preventative logic and precautionary logic lies in the role of uncertainty, a term, however, which we read as ignorance. We conclude from the discussing in [21] that formalizing applications of preventative logic lies within the scope of subjective probabilistic likelihood transfer mediated inference, while formalizing application of precautionary logic does not. Trouwborst insists that precautionary logic goes beyond the precautionary principle, precautionary logic being useful outside ecological areas just as well, while the precautionary principle would advocate additional warrant for the use of precautionary logic in an ecological setting. We follow this advice by not simply referring to PIR-likelihood-transfer as the precautionary principle. In [14] precautionary logic is use as a label for applications of the/a precautionary principle, and a survey is given of different perspectives on political pressures which the use of precautionary logic may surround.

Cameron & Peloso [11] use fuzzy logic to provide an analysis of precautionary reasoning which seems to be in essence the same as ours, though in a fairly different wording, and without much emphasis on subjective probabilities.

Resnik [16] provides a comprehensive though informal account of precautionary reasoning, which is more inclusive than mere applications of a precautionary inference rule. Resnik's account makes only marginal mention of subjective probabilities, and on p31 it is stated that:

> While the subjective approach may be suitable for personal decision-making that does not involve the public, such as placing bets on horse races, a strong case can be made that for public decision-making one should use probabilities that are as free as possible from biases because the stakes are much higher.

We take the above quote for a fundamental misunderstanding of subjec-

tive probability logic. Subjective probabilities can be used under complex conditions where meticulous quantitatively based probabilistic reasoning is imperative (such as DNA sample based arguments in forensics), and where other approaches are less defensible. Resnik, however, discusses other doubts concerning PP, see e.g. [15].

Beyleveld & Brownsward [9] describe a principle of precautionary rational reasoning (PRPR) which they suggest embodies precautionary reasoning in a best defensible form. The following quote suggests a discrepancy with our analysis. In the final sentence the change from scientific certainty to scientific uncertainty (in our terminology, the change from uncertainty to ignorance) may only strengthen the initial case for precaution in the presence of additional expert advice. When applying PIR-likelihood-transfer, the transfer of an ignorance interval followed by oppositional parameter determination (which need not be provided with fitting expert advice) may strengthen the case for an intervention which otherwise might not be put into effect.

> First, there is no obvious improvement in the argument for precaution when the context changes from one of scientific certainty to one of scientific uncertainty. Suppose a situation of scientific certainty: the experts agree, let us suppose, that the use of mobile phones causes headaches. Before responsible regulators take precautionary measures, they will count the cost of giving up or restricting the use of mobiles. Let us suppose that regulators decide against a precautionary intervention; the benefit of mobiles is high, and headaches are a minor problem. If regulators so decide in a context of expert agreement, it is not obvious why a change to scientific uncertainty should improve the case for precautionary intervention—it will only do so if some experts now suggest that the harm to users is more serious than the occasional headache. Similarly, if regulators initially decide in favour of precautionary intervention, the change from scientific certainty to uncertainty does not necessarily strengthen the initial case for precaution—again, it will do so only if some experts now suggest that the harm to users is more serious than the occasional headache.

Beyleveld & Pattinson [10] describe a purely philosophical application of PRPR (then not under than name) which is unconnected with environmental

issues. Said form of PRPR agrees with an application of the minimax principle which is rational in the presence of very high meta-uncertainty. The rule PIR-likelihood-transfer may be understood as a variation on the minimax principle where an opponent (OMOE) is asked to indicate a conceivable bad outcome that must be prevented, although the result may be that a course of action is chosen which would have been outperformed by a true (though in practice unfeasible) minimax strategy.

Van Asselt & Vos [22] indicate the presence of a paradox in connection with the precautionary principle. In the terminology of our paper the paradox reads thus: on the one hand (scientific) ignorance is required to invoke an application of precautionary inference while on the other hand science is needed to demonstrate the possibility of the hazard which (though with a probability that is relatively unknown due to meta-uncertainty) must be avoided. PIR-likelihood-transfer avoids this paradox by downplaying the need for a plausibility demonstration (i.e., the oppositional parameter selection requires no further motivation). By avoiding the paradox, however, we run the risk that the rule PIR-likelihood-transfer is too strong and generates undesirable results too often.

Dinneen [12] provides additional terminology, we mention: (i) precaution simple: involves pragmatic approaches to precaution; (ii) precautionary approach: the terminology is confusing, as a primary source of the precautionary principle, the Rio Declaration on the Environment and Development, speaks of a precautionary approach, the core of which seems only accidentally to have been labeled as a principle; (iii) principled precaution (which may or may not involve that application of a specific precautionary logic); (iv) precautionary discourse: arguments about precaution including reflection about uses of precautionary principles.

The theory of existential risks, see for instance Torres [20] is still almost disjoint with precautionary discourse. However, it is plausible that precaution, in a form which can only be delivered by applications of precautionary inference, is the only way to deal with existential risks, if there is any way at all.

6.3 Precautionary Logic and Accusations

We understand accusations in the perspective of accusation theory as developed in [5, 6] in the same style as the promise theory of [4]. A remarkable multitude of accusations may come with a single application of precautionary inference:

(i) that there were no grounds to adopt a precautionary principle, and to apply a precautionary inference rule in the given context;

(ii) that the precautionary inference rule has been misapplied (wrong rule chosen, rule wrongly applied, preconditions left unchecked);

(iii) that overprecaution has resulted from best intentions;

(iv) that precautionary inference has been applied in order to avoid an accusation of carelessly and without need exposing a part of the world to potentially unacceptable risks;

(v) that an invalid choice has been made between moderate precaution (viewing precaution as an important consideration) and so-called aggressive precaution (reversing the burden of proof);

(vi) that the use of precautionary inference is primarily motivated by lack of trust in promises.

There is a vast literature with criticism on "the precautionary principle". The difficult is to distinguish valid and convincing applications of precautionary logic from problematic applications of precautionary reasoning, in order to answer to accusations as listed above.

6.4 Connection with Criminal Law Cases

Looking at court procedures from the perspective of entropy and uncertainty is somehow counterintuitive. As is explained in some detail in [2] a court process in a penal case may be modeled as follows: the trier of fact, TOF in the notation of [2], which may be a judge, a team of judges, or a jury) starts out with a probability distribution $P_{TOF,initial}$ representing their prior odds including an assertion G_d representing that the defendant d is guilty of a specified crime. For the prior odds of TOF it is plausible that G is quite low say $P_{TOF,initial}(G_d) = 1/1.000.000$, or even much less because there is no initial assumption of guilt, while Bayesian reasoning requires TOF to start with non-trivial probabilities (in other words there must be some uncertainty).

Then in a number of steps on the basis of processing incoming information $P_{TOF,initial}$ is transformed (with transformations surveyed in [2]) to a final posterior distribution $P_{TOF,final}$. Incoming information may be

provided by the prosecutor as coming from police investigations, or from forensic experts playing the MOE (mediator of evidence) role.

Finally once all incoming information has been processed and the TOF has made up their mind by maintaining P_{final}, a threshold p_{thr} is assumed, say $p_{thr} = 4/5$. Now there are two cases: if $P_{final}(G_d) \geq p_{thr}$ then the defendant is considered guilty. In the second case, i.e, if $P_{final}(\neg G_d) \geq p_{thr}$, the defendant is considered to have been conclusively freed from the suspicion of being guilty, otherwise the case is concluded by acquittal for the defendant on the basis of lack of evidence available to the court.

The remarkable, and at first sight counterintuitive, observation is that in the second case, say assuming that $P_{final}(\neg G) = 5/6$, the uncertainty of TOF has in fact increased instead of decreased. If however TOF would maintain a belief function involving a proposition G_e for all other agents who might conceivably come into question for being considered (so that the sum over all e, including d of G_e equals 1) then the increase of uncertainty would be undone, and with essential the same session (though with many nearly redundant propositions G_e for $e \neq d$ included) d would be convicted while uncertainty may decrease as the increase in uncertainty for d is compensated by the decrease of uncertainty about the many other potential candidates for conviction.

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